1.1 p 120

a)

$f(x) = \dfrac{1}{(1+x)^2}$ sur $]-\infty\ ;-1[$ de la forme $\dfrac{u'(x)}{u(x)^2}$

$F(x) = -\dfrac{1}{1+x}$

c) $h(x) = \sin x \cos x$ de la forme $u'(x)\ u(x)$ avec $u(x) = \sin x$

$H(x) = \dfrac{1}{2}\sin^2 x$

ou encore en prenant $u(x) = \cos x$

$H(x) = -\dfrac{1}{2}\cos^2 x$

1.2 p 120

a- $f(x) = \dfrac{1}{x^2} - \dfrac{1}{\sqrt{x}} = -(-\dfrac{1}{x^2}) - 2\dfrac{1}{2\sqrt{x}}$

$F(x) = -\dfrac{1}{x} - 2\sqrt{x}$

b- $f(x) = \dfrac{\sin x}{\sqrt{\cos x + 2}} = -\dfrac{-\sin x}{\sqrt{\cos x + 2}} = -\dfrac{u'(x)}{\sqrt{u(x)}}$

$F(x) = -\sqrt{u(x)} = -\sqrt{\cos x + 2}$

c- $f(x) = \dfrac{(\sqrt{x}+1)^2}{\sqrt{x}} = 2\dfrac{1}{2\sqrt{x}} \times (\sqrt{x}+1)^2 = 2u'(x)u^2(x)$

$F(x) = u^3(x) = \dfrac{2}{3}(\sqrt{x}+1)^3$

2.2 p 121

$f(x) = \dfrac{-2x^3 + 3x^2}{(x-1)^2} = \dfrac{-2x(x-1)^2 - x^2 + 2x}{(x-1)^2} = \dfrac{-2x(x-1)^2 - (x-1)^2 + 1}{(x-1)^2} = -2x - 1 + \dfrac{1}{(x-1)^2}$

$F(x) = -x^2 - x - \dfrac{1}{x-1}$

#6 p 123

a- $f(x) = 1 - x + x^2 - x^3$ et $F(1) = 0$

$F(x) = x - \dfrac{1}{2}x^2 + \dfrac{1}{3}x^3 - \dfrac{1}{4}x^4 + k$

$F(1) = 1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + k = \dfrac{12-6+4-3}{12} + k = \dfrac{7}{12} + k$ d'où $k = -\dfrac{7}{12}$

b- $f(x) = -2\sin 2x$ et $F(\dfrac{\pi}{4}) = 1$

$F(x) = \cos 2x + k$

$F(\frac{\pi}{4}) = \cos\frac{\pi}{2} + k = k = 1$

F(x) = cos $2x$ + 1

c- $f(x)$ = cos $3x$ et $F(\frac{\pi}{2}) = 0$

$F(x) = \frac{1}{3}\sin 3x + k$

$F(\frac{\pi}{2}) = \frac{1}{3}\sin\frac{3\pi}{2} + k = -\frac{1}{3} + k = 0$

$F(x) = \frac{1}{3}\sin 3x + \frac{1}{3}$

d- $f(x) = x + \frac{1}{x^2} - \frac{1}{\sqrt{x}} = x - (-\frac{1}{x^2}) - 2\frac{1}{2\sqrt{x}}$ et F(1) = 1

$F(x) = \frac{1}{2}x^2 - \frac{1}{x} - 2\sqrt{x} + k$

$F(1) = \frac{1}{2} - 1 - 2 + k = -\frac{5}{2} + k = 1$

$F(x) = \frac{1}{2}x^2 - \frac{1}{x} - 2\sqrt{x} + \frac{7}{2}$

#8 p 123

a- $f(x) = x^3 - 2x + 1$

$F(x) = \frac{1}{4}x^4 - x^2 + x$

b- $f(x) = x + \frac{1}{\sqrt{x}} = x + 2\frac{1}{2\sqrt{x}}$

$F(x) = \frac{1}{2}x^2 + 2\sqrt{x}$

c- $f(x)$ = sin x – 2 cos x

F(x) = – cos x – 2 sin x

d- $f(x) = \frac{1}{x^2} - x^2$

$F(x) = -\frac{1}{x} - \frac{1}{3}x^3$

e- $f(x) = 1 - \frac{1}{\cos^2 x}$

F(x) = x – tan x

f- $f(x) = \cos\frac{x-\pi}{4}$

$F(x) = 4\sin\frac{x-\pi}{4}$

#13 p 123 (Terracher)

a- $f(x) = 2x(x^2-1)^5$

On pose $u(x) = x^2 - 1$

$u'(x) = 2x$

La dérivée de la fonction $(x^2 - 1)^6$ est $6\times 2x\times(x^2 - 1)^5$

donc $F(x) = \frac{1}{6}(x^2-1)^6$

b- $f(x) = \frac{x}{(x^2+2)^2} = \frac{1}{2}\frac{2x}{(x^2+2)^2} = \frac{1}{2}\frac{u'(x)}{u^2(x)}$

$F(x) = -\frac{1}{2}\frac{1}{u(x)} = -\frac{1}{2(x^2+2)}$

c- $f(x) = x\cos x^2 = \frac{1}{2}\times 2x\cos x^2 = \frac{1}{2}u'(x)\cos u(x)$

$F(x) = \frac{1}{2}\sin u(x) = \frac{1}{2}\sin x^2$

d- $f(x) = \sin 2x - \cos 2x = \frac{1}{2}2\sin 2x - \frac{1}{2}\cos 2x$

$F(x) = -\frac{1}{2}\cos 2x - \frac{1}{2}\sin 2x$

#32 p 125

a- $f(x) = x^2(x^3+2)^3 = \frac{1}{3}3x^2(x^3+2)^3 = \frac{1}{3}u'(x)u^3(x) = \frac{1}{3}\times\frac{1}{4}\times 4u'(x)u^3(x)$

$F(x) = \frac{1}{12}u^4(x) = \frac{1}{12}(x^3+2)^4$

b- $f(x) = \frac{x^2}{(x^3+2)^3} = x^2(x^3+2)^{-3} = \frac{1}{3}3x^2(x^3+2)^{-3} = \frac{1}{3}\times\frac{1}{-2}\times(-2)3x^2(x^3+2)^{-3}$

$F(x) = -\frac{1}{6}(x^3+2)^{-2} = -\frac{1}{6(x^3+2)^2}$

#33 p 125

a- $f(x) = \frac{x^3}{\sqrt{1+x^4}} = \frac{1}{4}\frac{4x^3}{\sqrt{1+x^4}} = \frac{1}{4}\frac{u'(x)}{\sqrt{u(x)}} = \frac{2}{4}\frac{u'(x)}{2\sqrt{u(x)}}$

$F(x) = \frac{1}{2}\sqrt{u(x)} = \frac{1}{2}\sqrt{1+x^4}$

b- $f(x)=\dfrac{\cos\sqrt{x}}{\sqrt{x}}=2\times\dfrac{1}{2\sqrt{x}}\times\cos\sqrt{x}=2u'(x)\cos u(x)$

$F(x)=2\sin u(x)=2\sin\sqrt{x}$

#34 p 125

a- $f(x)=\dfrac{x}{(1+x^2)^2}=\dfrac{1}{2}\dfrac{2x}{(1+x^2)^2}=\dfrac{1}{2}\dfrac{u'(x)}{u^2(x)}$

$F(x)=-\dfrac{1}{2}\dfrac{1}{u(x)}=-\dfrac{1}{2(1+x^2)}$

b- $f(x)=\dfrac{x-1}{\sqrt{x^2-2x+3}}=\dfrac{1}{2}\dfrac{2(x-1)}{\sqrt{x^2-2x+3}}=\dfrac{1}{2}\dfrac{2x-2}{\sqrt{x^2-2x+3}}=\dfrac{u'(x)}{2u(x)}$

$F(x)=\sqrt{u(x)}=\sqrt{x^2-2x+3}$

n°39

$f(x)=\dfrac{3}{x^2}\left(\dfrac{x-1}{x}\right)^2$ avec $u(x)=\dfrac{x-1}{x}=1-\dfrac{1}{x}$ et $u'(x)=\dfrac{1}{x^2}$

$f(x)=3u'(x)u^2(x)$

$f(x)=\left[u^3(x)\right]'$

donc

$F(x)=\left(\dfrac{x-1}{x}\right)^3$

p 125 n° 42

$f(x)=\dfrac{3x^2+12x-1}{(x+2)^2}=\dfrac{3(x+2)^2-13}{(x+2)^2}=3-\dfrac{13}{(x+2)^2}$

soit $g(x)=\dfrac{13}{(x+2)^2}$ on pose $u(x)=x+2$ et $u'(x)=1$

$g(x)=-13\dfrac{-u'(x)}{u^2(x)}=-13\left(\dfrac{1}{x+2}\right)'$ on obtient $G(x)=-\dfrac{13}{x+2}$

on a donc $F(x)=3x+\dfrac{13}{x+2}+k$

p 125 n°45

$f(x)=\dfrac{8x}{\left(x^2-4\right)^2}$

mettons f(x) sous la forme : $\dfrac{a}{(x-2)^2}+\dfrac{b}{(x+2)^2}$

$f(x)=\dfrac{ax^2+4ax+4a+bx^2-4bx+4b}{\left(x^2-4\right)^2}=\dfrac{x^2(a+b)+x(4a-4b)+4a+4b}{\left(x^2-4\right)^2}$

par identification, on a $a+b=0$ ou $a=-b$

et $4a-4b=8$

$-4b-4b=8$

$-8b=8$ $d'ou$ $b=-1$ et $a=1$

donc $f(x)=\dfrac{1}{(x-2)^2}-\dfrac{1}{(x+2)^2}$

en posant $u(x)=x-2$ avec $u'(x)=1$

et $v(x)=x+2$ avec $v'(x)=1$

on a : $f(x)=\dfrac{-u'(x)}{u^2(x)}-\dfrac{v'(x)}{v^2(x)}$

$d'ou$ $F(x)=-\dfrac{1}{(x-2)}+\dfrac{1}{(x+2)}+k$